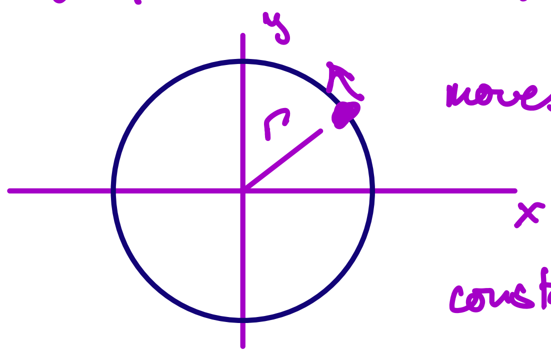
Previous chapter covered oscillations

review: uniform circular motion



moves along circle

constant radius r

tangential velocity v_t constant magnitude

$v_t = \frac{\text{dist}}{\text{time}}$ let T = period time for 1 complete revolution

dist 1 rev = $2\pi r$ circumf of circle

so $\boxed{v_t = \frac{2\pi r}{T}}$

define frequency = # rev/sec

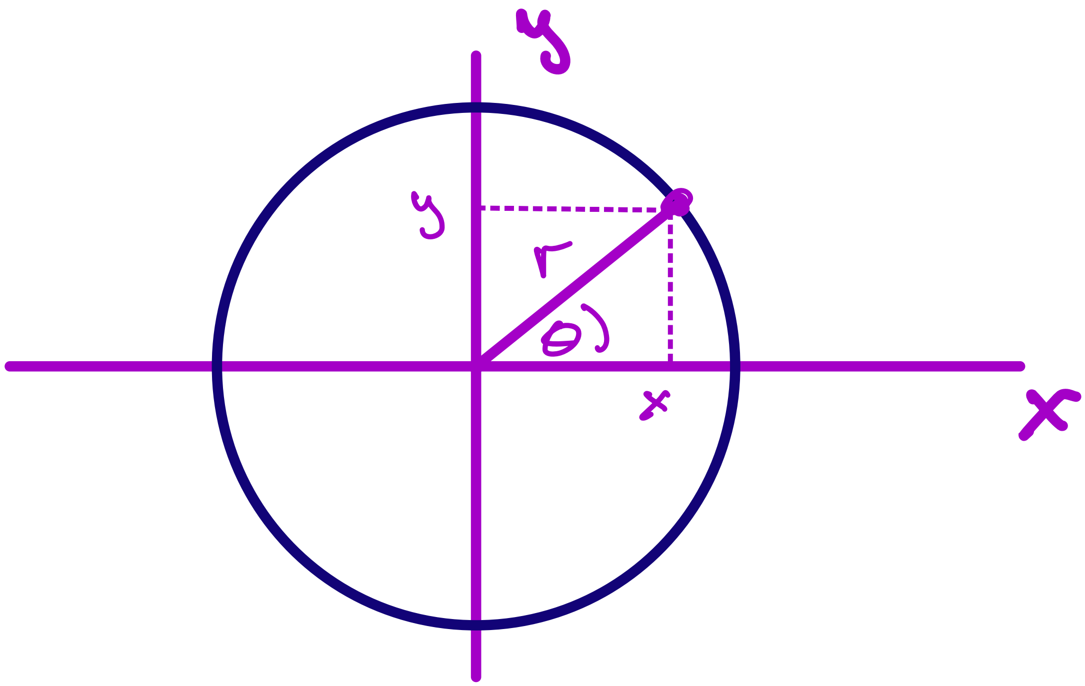
$\boxed{f = \frac{1}{T}}$ 1 rev in time T

so define angular frequency ω = # angles/sec

$\boxed{\omega = \frac{2\pi}{T} = 2\pi f}$

and $\boxed{v_t = \omega r}$

what does motion along x & y look like?



$x = r\cos\theta$
$y = r\sin\theta$ } θ is changing uniform rate

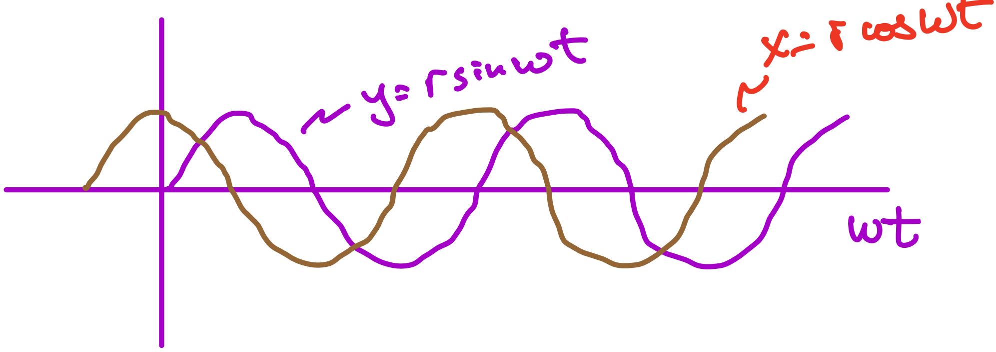
that means $\frac{d\theta}{dt} = \text{const} = \frac{\Delta\,\text{angle}}{\Delta\,\text{time}} = \frac{2\pi}{T} = \omega$

solve $\frac{d\theta}{dt} = \omega$ constant

$\theta = \omega t$ ($\theta = 0$ at $t = 0$)

so $x = r\cos\omega t$

$y = r\sin\omega t$



x & y exhibit simple harmonic motion

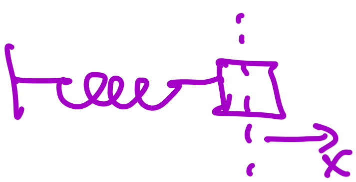
$x = \underset{\text{amplitude}}{r}\cos\underbrace{\omega t}_{\text{always an angle = "phase"}}$ SHM

more general : $x = r\cos(\omega t + \underset{\text{phase constant}}{\phi})$

ϕ just tells you where x is when $t=0$

$\Rightarrow$ something controlled by initial conditions

x can measure position around circle



displacement from equilibrium

if x measures displ from equilibrium, then it takes energy to displace it

usually $E \propto x^2$ (energy $\propto$ amplitude^2)

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Waves

oscillations can generate waves

ex: oscillations in bath tub generates water waves

waves are a "disturbance" in a medium

water waves: disturbance in water with gravity and inertia

sound: pressure disturbance propagating in air

EM: electromagnetic disturbance moving through space - but no medium (special case!)

Disturbance propagates along medium:

transverse: disturbance $\perp$ wave direction
e.g. water waves

longitudinal : disturbance parallel to wave direction

e.g. sound

earthquakes produce both !

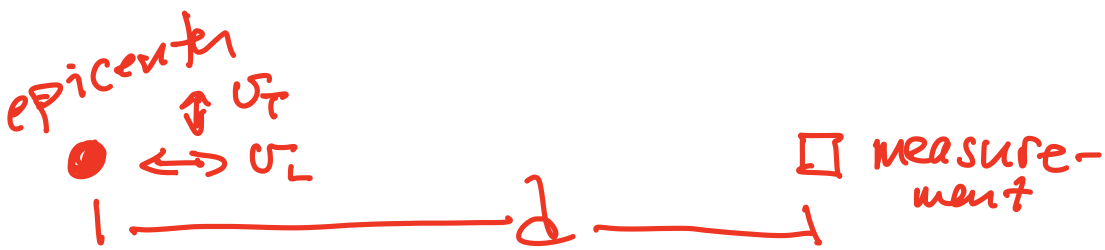
note: wave propogation velocity will depend on properties of medium

$\Rightarrow$ so v_t for transverse

v_L for longitudinal

for earthquakes $v_L > v_t$

$\Rightarrow$ so can use this to locate epicenter by measuring time diff between v_t & v_L in 3 different locations (knowing v_t & v_L)



• an earthquake happens at some location and at time t_0

- an unknown distance d away are seismometers that can measure transverse & longitudinal waves
- v_t vel of transverse wave
  v_L " " longitudinal wave
  $v_L > v_t$
- seismometers detect waves at times t_L & t_t

$d = v_t(t_t - t_0) = v_L(t_L - t_0)$

$\Delta t = t_t - t_L$ time between arrival of the 2 waves

use $t_t = \frac{d}{v_t} + t_0$

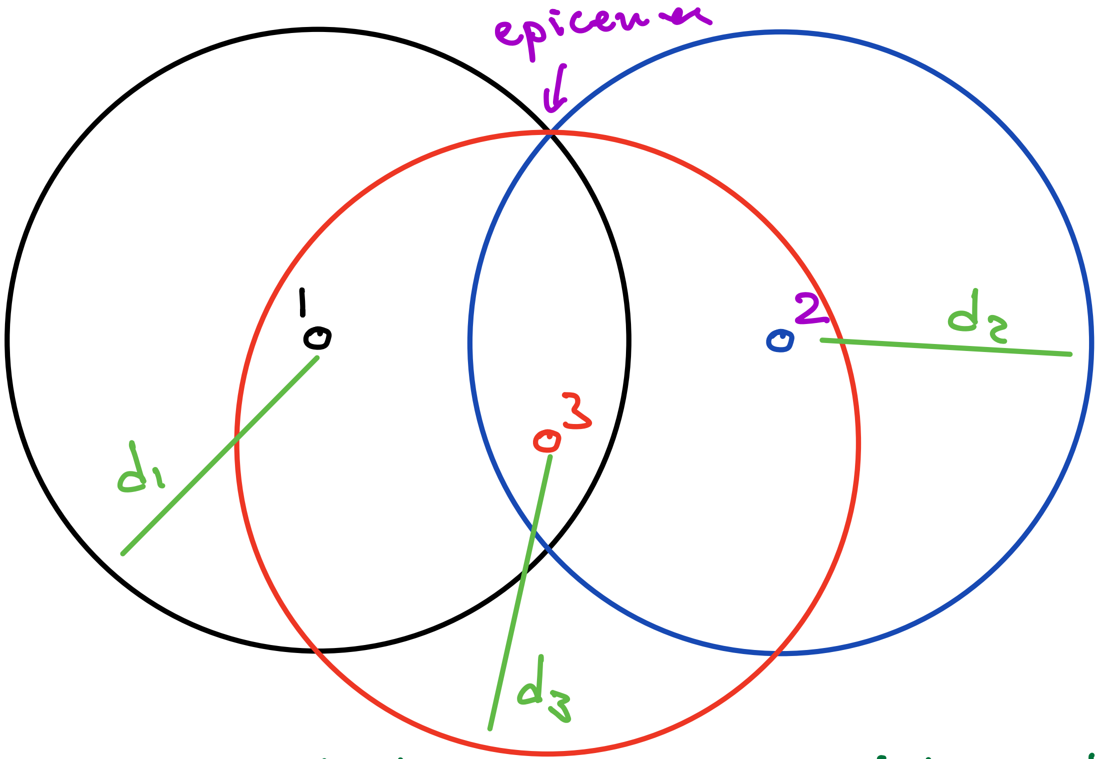
$t_L = \frac{d}{v_L} + t_0$

$\Delta t = \frac{d}{v_t} + t_0 - \left(\frac{d}{v_L} + t_0\right)$

$= \frac{d}{v_t} - \frac{d}{v_L} = d\left(\frac{1}{v_t} - \frac{1}{v_L}\right)$

so $d = \dfrac{\Delta t}{\frac{1}{v_t} - \frac{1}{v_L}} = \dfrac{v_t \Delta t}{1 - \frac{v_t}{v_L}}$

- by measuring Δt and kowing v_t, v_L can calculate d



measure Δt at point 1, 2, 3 and triangulate

Mechanical waves

parameters

Period: T time for 1 cycle

Frequency: f # cycles/sec

$\Rightarrow$ 1 cycle so $f = 1/T$

Angular frequency: $\omega = \frac{2\pi}{T} = 2\pi f$

Amplitude: max displacement

ex: $x = r \cos \omega t$

amplitude

angular frequency

wave propagates in some direction
with velocity v

v depends on medium

ex: sound $\Rightarrow$ pressure disturbance in a medium

air	water	rock	steel
343 m/s	1500 m/s	2000 m/s	6000 m/s

in general: denser medium $\Rightarrow$ higher velocity for
pressure waves

why? because wave is from interplay between
inertia and restoring forces

water: gravity

spring: spring elastic properties

wavelength: λ

defined as distance wave repeats

- if wave propagates w/ velocity v
- and wave repeats every T seconds

then $\boxed{\lambda = vT}$

or $\lambda = \frac{v}{f}$ so $\boxed{v = \lambda f}$

this is a general characteristic of mechanical waves

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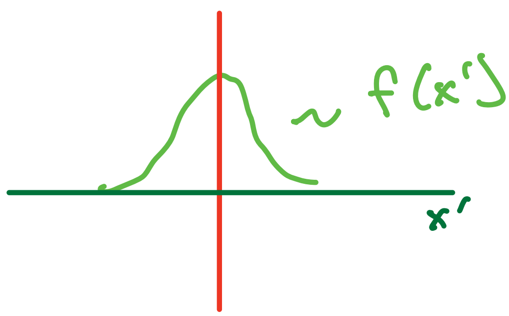
Wave pulse see https://www.physics.umd.edu/hep/drew/shiva_wave.html#traveling

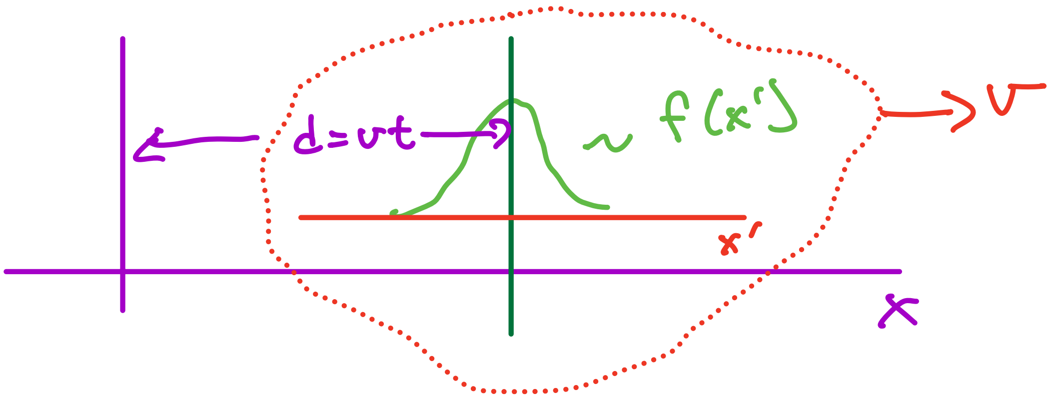
assume shape stays constant as wave moves

in rest frame of wave let x' be coordinate relative to wave center



is constant in wave reference frame

in lab frame wave moves along x



frame x' moves dist $d = vt$ in time t

so $x = d + x' = vt + x' \Rightarrow x' = x - vt$

so $f(x') = f(x - vt)$

this is the "wave function" of the wave in lab reference frame

in general: $f(x - vt)$ wave moves in $+x$ dir

$f(x + vt)$ " " " $-x$ "

$(v \to -v)$

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Sinusoidal traveling wave: $f \to \cos$ or $\sin$

see https:www.physics.umd.edu/hep/drew/shiva_wave.html#periodic

note: trig functions argument are angles!

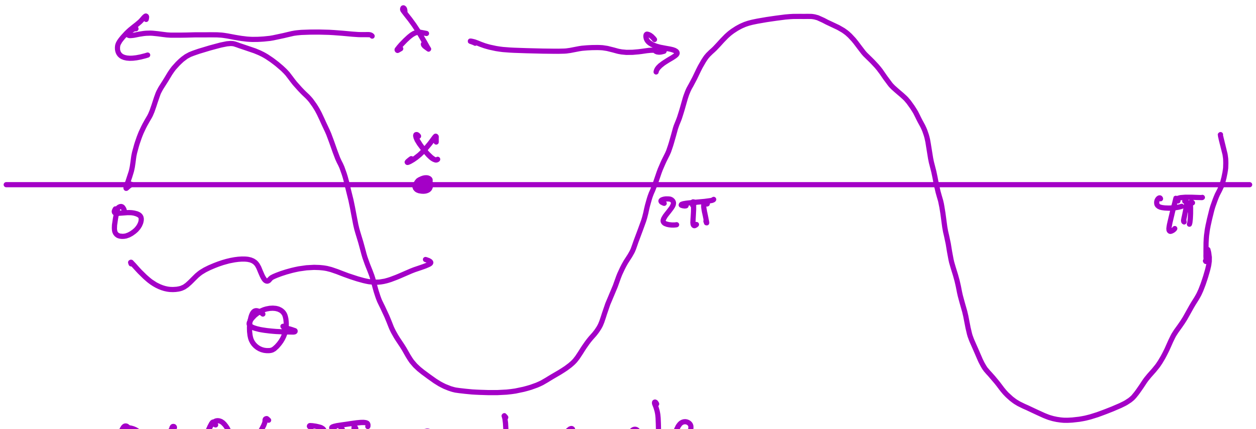
if wave moves dist λ in 1 period then

x goes through 2π in 1λ

4π in 2λ

$\vdots$

so angle is $x \cdot \frac{2\pi}{\lambda}$



$0 \leq \theta \leq 2\pi$ each cycle

for sinusoidal wave (sin or cos):

$y(x', t) = A\cos\left(x' \cdot \frac{2\pi}{\lambda}\right)$

so $y(x,t) = A\cos\left[\frac{2\pi}{\lambda}(x - vt)\right]$

write $k \equiv \frac{2\pi}{\lambda}$ "wave number"

also $\frac{2\pi v}{\lambda} = \frac{2\pi}{\lambda} \cdot \lambda f = 2\pi f = \omega$

so can write $y(x,t) = A\cos(kx - \omega t)$

wave moving in $+x$ dir

$= A\cos(kx + \omega t)$

wave moving in $-x$ dir

note: $v = \lambda f = \frac{\lambda}{2\pi} \cdot 2\pi f = \frac{\omega}{k}$

$$\boxed{v = \frac{\omega}{k}}$$

if you use cos, then $y = A$ when $x=0, t=0$

this your initial condition!

if instead your initial condition was

$y = 0$ at $x=0, t=0$

then $y(x,t) = A \sin(\underbrace{kx - \omega t}_{\text{"phase"}})$

note: you can be general and add a

"phase constant" ϕ

$y(x,t) = A \sin(kx - \omega t + \phi)$

ϕ is a function only of initial conditions

$x=0, t=0$, $y(0,0) = A \sin\phi$

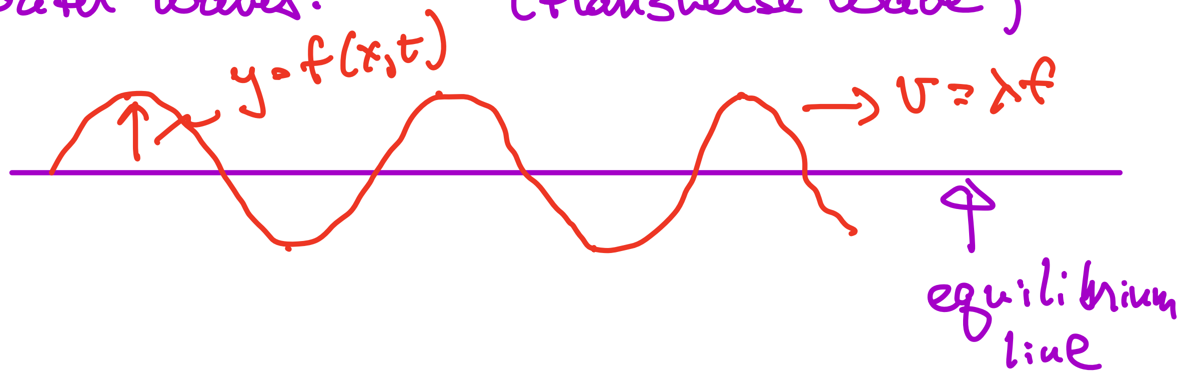
if $\phi = 0$, $y(0,0) = 0$

if $\phi = \pi/2$, $y(0,0) = A$

and somewhere in between

Velocity & accel of medium

water waves: (transverse wave)



wave disturbance propagates with velocity $v = \lambda f$

but medium only moves up & down dist y

for medium $y = A \sin(kx - \omega t + \phi)$

vel of medium $v_y = \frac{\partial y}{\partial t} = -A\omega \cos(kx - \omega t + \phi)$

$= -v_{max} \cos(\quad)$

$v_{max} = A\omega$

velocity amplitude

accel $a_y = \frac{\partial v_y}{\partial t} = -A\omega^2 \sin(kx - \omega t + \phi)$

$= a_{max} \sin(\qquad)$

$a_{max} = A\omega^2$ accel amp

note: $a_y = A\omega^2 \sin(\quad) = -\omega^2 y$

so $\frac{\partial^2 y}{\partial t^2} = -\omega^2 y$

take $\frac{\partial^2 y}{\partial x^2} = \frac{\partial}{\partial x} \frac{\partial y}{\partial x} = \frac{\partial}{\partial x}(-Ak \sin(\quad))$

$= -Ak^2 \sin(\quad) = -k^2 y$

$$\left.\begin{aligned} &\text{so} \quad \frac{\partial^2 y}{\partial x^2} = -k^2 y \\ &\text{and} \quad \frac{\partial^2 y}{\partial t^2} = -\omega^2 y \end{aligned}\right\} \text{ eliminate } y$$

$\frac{1}{k^2} \frac{\partial^2 y}{\partial x^2} = \frac{1}{\omega^2} \frac{\partial^2 y}{\partial t^2}$

or $\frac{\partial^2 y}{\partial x^2} - \frac{k^2}{\omega^2} \frac{\partial^2 y}{\partial t^2} = 0$ but $v = \omega / k$

so $\frac{\partial^2 y}{\partial x^2} - \frac{1}{v^2} \frac{\partial^2 y}{\partial t^2} = 0$

this is the linear wave equation and is true for any 1-D linear wave

"linear" means no terms like $\frac{\partial^2 y}{\partial x^2} \cdot x$ etc

linear gives us "principle of superposition"

ex: $y_1 = f_1(x,t)$
$y_2 = f_2(x,t)$ } 2 waves (like from dropping 2 stones in diff positions on lake

resulting wave $y_{tot} = y_1 + y_2$ linear!

<u>Velocity of waves in medium</u>

waves come from interplay between inertial and restoring forces

inertial: ma so involve mass
restoring: depends on medium

water: gravity
air: compressibility of air
strings tension in string

these forces imparts energy into wave

($KE \propto v^2$)

so guess $v^2 \propto \frac{\text{restoring}}{\text{inertial}}$

increase this and disturbance propagates faster

increase this and its harder to disturb medium

so we guess $v \propto \sqrt{\frac{F_{restore}}{\text{inertial mass}}}$

to get units right: mv^2 = units of energy
= Force × dist

so $v \propto \sqrt{\frac{F_{restore} \cdot dist}{mass}}$

$v = \sqrt{\frac{F_{restore}}{mass/dist}}$ for string

μ = mass/dist   mass per length of string
$F_{tension}$ = tension of string

$$v = \sqrt{T/\mu} \quad \text{string}$$

ex: guitar string mass ~ 0.3g high E
~ 6.0g low E
they are around 25 in ~ 60cm = 0.6m

so $\mu = \frac{0.3g}{0.6m} = 0.5 g/m$ high E

$= \frac{6.0g}{0.6m} = 10 g/m$ low E

speed of wave $v \sim 450 m/s$ for high E

so $v = \sqrt{T/\mu}$

$\Rightarrow T = \mu v^2 = 0.5 g/m \cdot (450 m/s)^2$

$= 0.5 \times 10^{-3} \frac{kg}{m} * \left(450 \frac{m}{s}\right)^2$

$= 101 N \sim 22$ lbs tension

in fluids:

restoring force is "elastic" (ability to compress, stretch)

usually we use "Bulk modulus"

$$B = \frac{\text{pressure needed}}{\text{volume change}} = \frac{\Delta P}{\Delta V/V}$$

inertial: mass/volume $\equiv \rho$

units: $[B] = \frac{\text{Force}}{\text{area}} = \frac{kg\, m/s^2}{m^2} = \frac{kg}{m\, s^2}$

$[\rho] = \frac{\text{mass}}{\text{volume}} = \frac{kg}{m^3}$

want to combine to get units $v^2 \equiv \frac{m^2}{s^2}$

take $\left[\frac{B}{\rho}\right] = \frac{kg}{m\, s^2} \frac{1}{kg/m^3} = \frac{m^2}{s^2}$ ✓

$v_{fluid} = \sqrt{\frac{B}{\rho}}$ is correct

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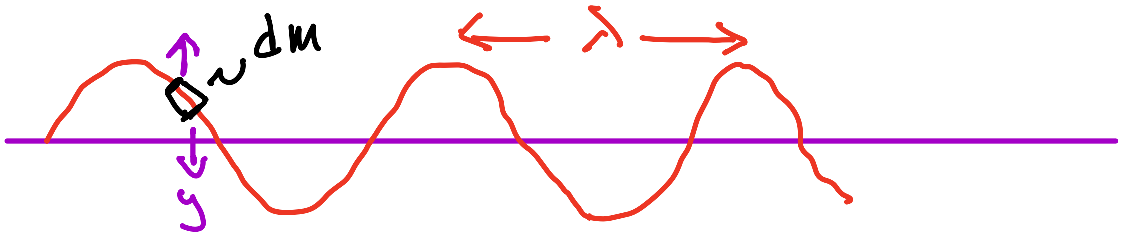
Energy in waves

take waves on oscillating string



Consider KE for 1 wavelength of string

$KE = \frac{1}{2} m_\lambda v^2$ $\quad m_\lambda$ is mass for 1 λ

if string has mass/length $\mu = \frac{\Delta m}{\Delta l}$

then $m_\lambda = \mu \lambda$

v^2 is average velocity of string

remember $y = A \sin(kx - \omega t)$

$v_y = -A\omega \cos(kx - \omega t)$

v^2_{ave} = is average over 1 period and 1 λ

Average of $f(x)$ over length L:

$$\bar{f} = \frac{1}{L} \int_0^L f(x)\,dx$$

so $v^2_{ave} = A^2\omega^2 \frac{1}{T}\int_0^T \left[\frac{1}{\lambda}\int_0^\lambda \cos^2(kx-\omega t)\,dx\right] dt$

do dx part 1st $\int_0^\lambda \cos^2(kx-\omega t)\,dx$

change of variables: $z = kx-\omega t$

$dz = k\,dx \Rightarrow dx = \frac{dz}{k}$

this integral becomes $\int_0^{2\pi} \cos^2(z)\frac{dz}{k}$

$= \lambda \underbrace{\int_0^{2\pi} \frac{\cos^2(z)\,dz}{2\pi}}$

this is average value of $\cos^2(z)$ over 1 period

$\cos^2(z)$ is between 0 & 1, good guess that average value is $\frac{1}{2}$

so $\int_0^\lambda \cos^2(kx-\omega t)\,dt = \frac{\lambda}{2}$

then $v^2_{av} = A^2\omega^2 \frac{1}{T}\int_0^T \left[\frac{1}{\lambda}\frac{\lambda}{2}\right] dt = \frac{1}{2}A^2\omega^2 \underbrace{\frac{1}{T}\int_0^T dt}_{1}$

so $v_{av}^2 = \frac{1}{2} A^2 \omega^2$

$$KE_\lambda = \frac{1}{2}\mu\lambda\left(\frac{1}{2}A^2\omega^2\right) = \frac{1}{4}A^2\omega^2\mu\lambda$$

Next: consider PE of stretching string

PE is the work done in stretching string by amount y

$\Rightarrow$ using spring model where force $F_s = -k_s$

k_s = spring constant

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this "spring" will oscillate w/ freq $\omega = \sqrt{\frac{k_s}{m_\lambda}}$

so write $k_s = \omega^2 m_\lambda$

$= \omega^2 \mu\lambda$

for spring, $PE = \frac{1}{2}k_s y^2$

PE_λ is average over 1λ, $1T$ just like above

$$PE_\lambda = \frac{1}{2}k_s \frac{1}{T}\int_0^T \frac{1}{\lambda}\int_0^\lambda y^2\,dx\,dt$$

$y = A\sin(kx - \omega t)$
and $\frac{1}{\lambda}\int_0^{\lambda} y^2\,dx = \frac{1}{2}$ as above

so $PE_\lambda = \frac{1}{2}\omega^2\mu\lambda A^2 \cdot \frac{1}{2} = \frac{1}{4}A^2\omega^2\mu\lambda$

same as KE_λ

total energy: $E_\lambda = PE_\lambda + KE_\lambda = \frac{1}{2}A^2\omega^2\mu\lambda$

Power

power $= \dfrac{\Delta E}{\Delta t}$

power averaged over 1 period T
is average energy per time T

$$P_{av} = \frac{E_\lambda}{T} = \frac{1}{2}A^2\mu\omega^2\frac{\lambda}{T}$$

$\frac{\lambda}{T} = \lambda f = v \leftarrow v$ along dir of wave propagation

so $P_{av} = \frac{1}{2}A^2\mu\omega^2 v$

$P_{av} \propto A^2$ & ω^2 and v

P_{av} is power of wave as it propagates along x-dir, so P_{av} over 1 λ is P_{av} of wave

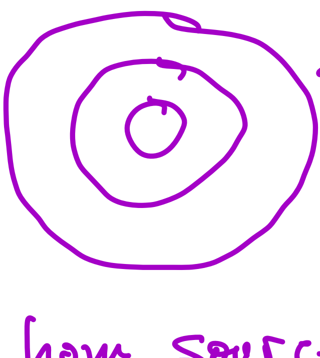
main concept: P_{av} wave $\propto A^2$ amplitude^2

$\Rightarrow$ true in general for all waves

wave Intensity

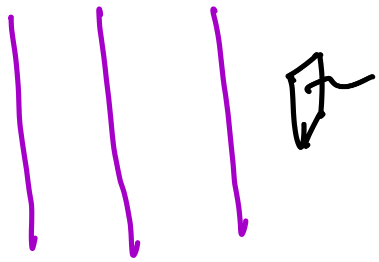
for 3-D wave (sound, radio, etc)

intensity is $\frac{P_{av}}{\text{area}}$



spherical waves

far from source wave looks like plane wave



area A

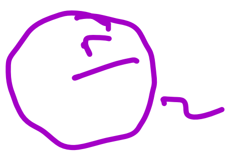
Intensity goes down as wave spreads out but P_{av} over wavefront is constant

$\Rightarrow$ so to calculate power flowing thru area A, use intensity

$$P_A = I \cdot A$$

$I \Rightarrow$ Power spread out over area of wave

Power $\Rightarrow$ watts $= \dfrac{\text{energy}}{\text{second}}$



area sphere $= 4\pi r^2$

Intensity is power smeared out over surface

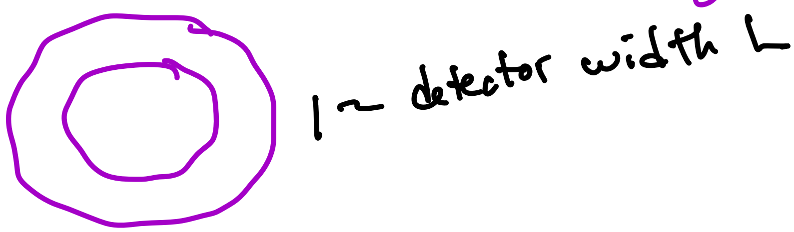
so $I = \dfrac{P_{av}}{4\pi r^2}$

and power on area A : $P_A = \dfrac{P_{av}}{4\pi r^2} \cdot A$

for 2-D wave: wave ripple in Pond

$I = \frac{P_{av}}{2\pi r}$ power per length

$2\pi r$ ~ circumference of circle



P_L power through L

$P_L = I \cdot L = \frac{P_{av}}{2\pi r} \cdot L$

ex: Sun's energy is in electromagnetic waves (covered more later this semester)

- intensity radiated decreases as you get further from sun
- at earth, $I \sim 1400 \frac{W}{m^2}$

$R_{earth-sun} = 93 \times 10^6 \, mi * 5280 \frac{ft}{mi} * \frac{1m}{3.281 \, ft}$

$= 150 \times 10^6 \, km$

total Power: $I = \frac{P_{av}}{area} \Rightarrow P_{av} = I \cdot area$

sun power output
$P_{sun\ ave} = 1400 \frac{W}{m^2} * 4\pi R_{earth\text{-}sun}^2$

$= 1400 \frac{W}{m^2} * 4\pi \times (150 \times 10^9 m)^2$

$= 1400 * 4\pi \times 150^2 \times 10^{18} W$

$= 395 \times 10^6 W * 10^9 * 10^9$

$= 395 \times 10^6 GW$ 1GW = decent size nuclear reactor

$I_{sun} = 1400 \frac{W}{m^2}$ at top of atmosphere

$\Rightarrow$ atmosphere absorbs $\rightarrow \sim 1000 \frac{W}{m^2}$ at surface

For 1 day, the energy absorbed would be

$I_{day} = 1 \frac{kW}{m^2} * \frac{12 hr}{day} = 12 \frac{kW}{m^2} \text{-} \frac{hrs}{day}$ 12 hrs daylight

$= 12 \frac{kWh}{m^2}/day/day$

$\Rightarrow$ light changes over the day & reduces this total

assume: summer $\Rightarrow$ average is $\sim \frac{1}{2}$ peak

winter $\Rightarrow$ " " $\sim \frac{1}{4}$ peak

so at our latitude, assume $\bar{I}$

summer : $\bar{I}_s = \frac{1}{2} \cdot I_{day} = 6 kWh/m^2/day$

winter : $\bar{I}_w = \frac{1}{4} I_{day} = 3 kWh/m^2/day$

Use solar panels to collect energy from sun

1. assume area is $\sim 1.6\,m^2$
2. " panels only convert $\sim 20\%$ of light to electric power (typical are 15–25%)

so efficiency $\varepsilon = 20\%$

so power $\bar{P}_{in} = \bar{I} \cdot A$ into panel

power converted to electricity

$$\bar{P}_{out} = \bar{P}_{in} \cdot \varepsilon$$

summer:

$$\bar{P}_s = 6\,\frac{kWh}{m^2 \cdot day} * 1.6\,m^2 * 20\% \sim 2\,\frac{kWh}{day} \text{ per panel}$$

winter:

$$\bar{P}_w = \frac{1}{2}\bar{P}_s = 1\,\frac{kWh}{day} \text{ per panel}$$

let's design for winter

Average home uses ~ 1 kW averaged over day

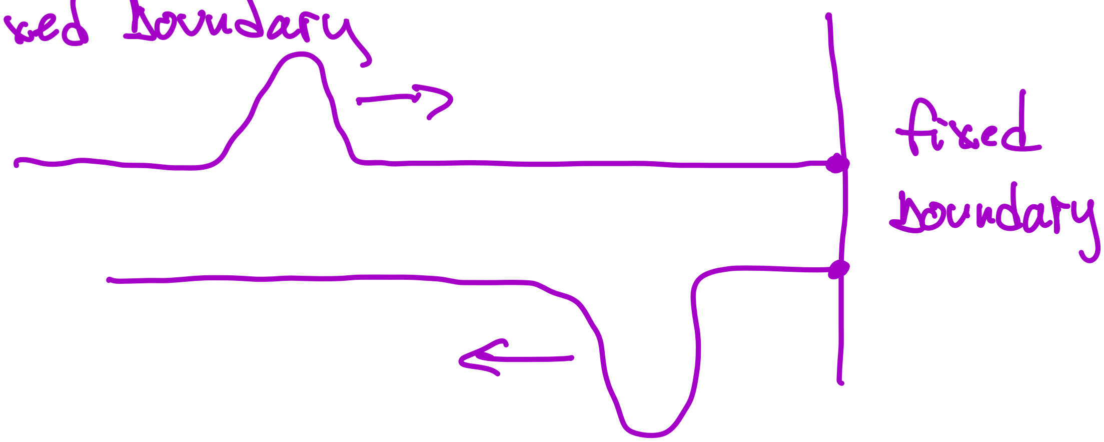
$$\bar{P} = 1\,kW * \frac{24\,hr}{day} = 24\,kWh/day$$

so we need 24 panels (at least)

Actually we also need batteries to charge during the day!

Reflection & Transmission at boundary

Fixed boundary



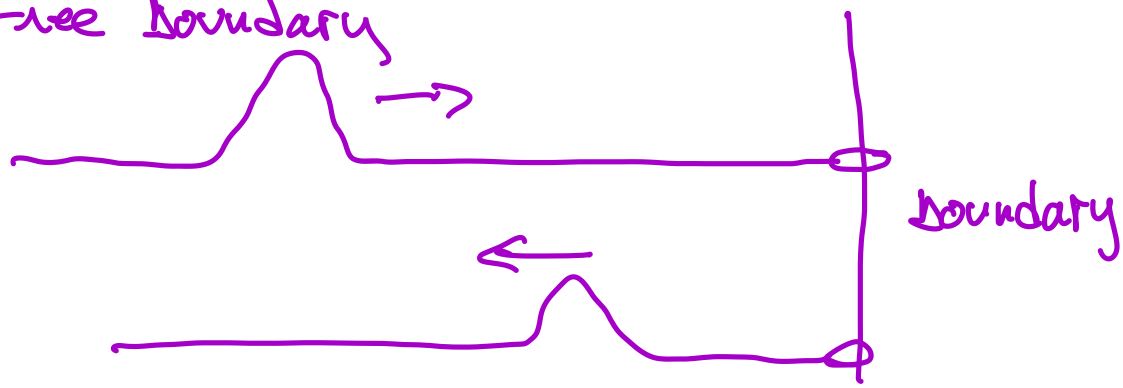
inverted at boundary due to Newton's 3rd law (equal & opposite forces) boundary exerts downward force to cancel wave's upwards force

=> can think of this situation as a wave in a medium that increased in "density"

Rule: if medium "density" increases:

- reflected pulse is inverted
- has a phase change of 180° (π)

Free boundary



no reaction force at boundary

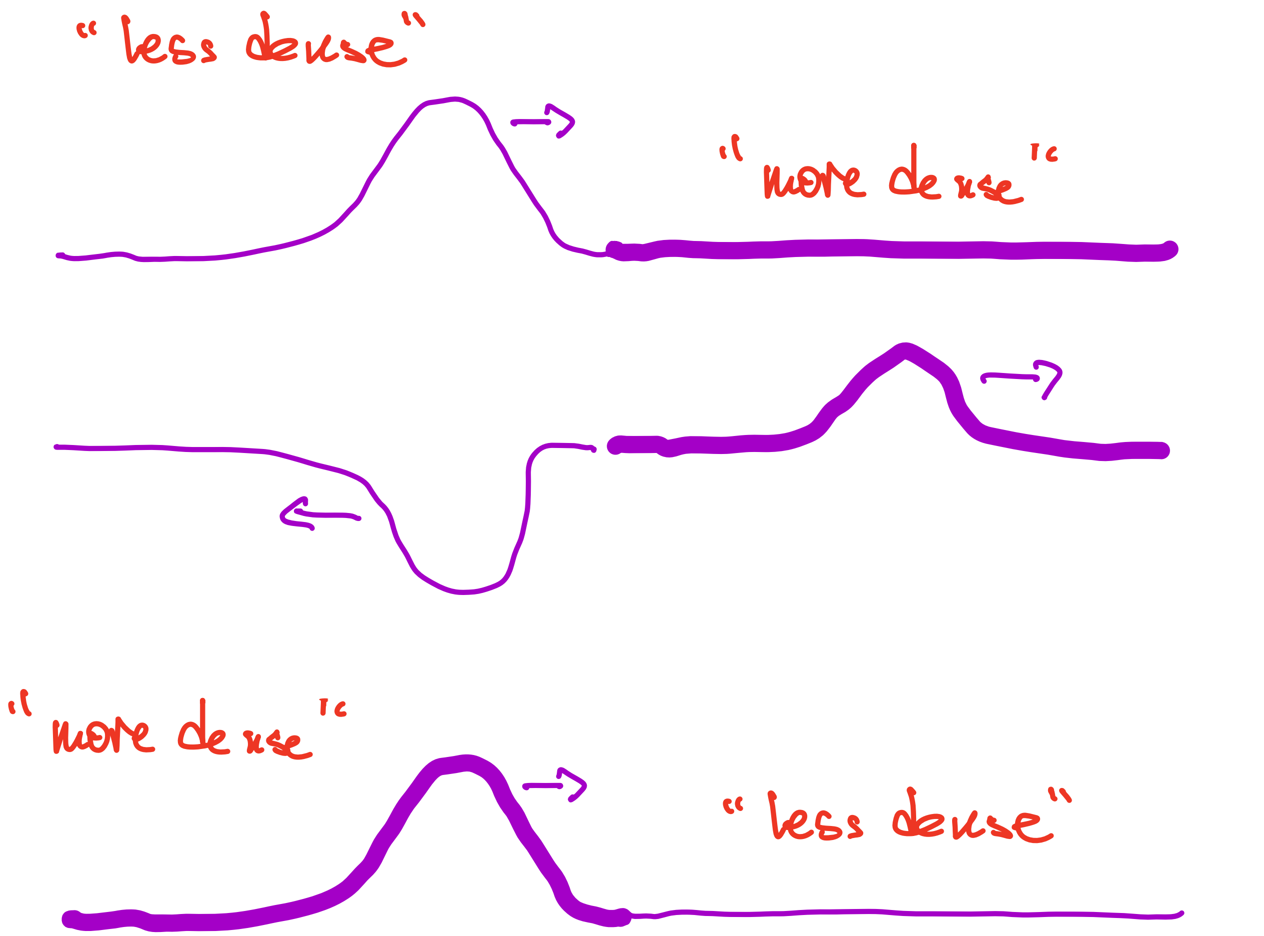
Rule: if medium "density" decreases:

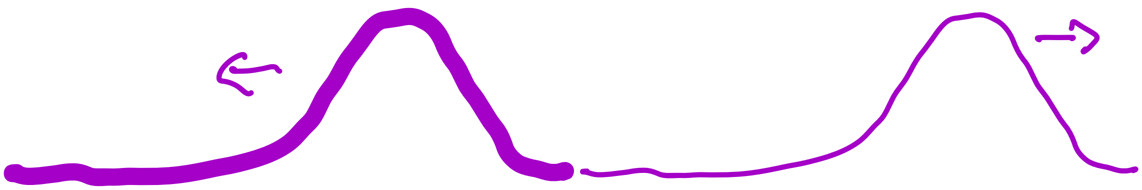
- reflected pulse is <u>NOT</u> inverted
- has no phase change of

Can have both reflection <u>AND</u> transmission

ex: light on glass   96% transmitted
                      4% reflected

ex: wave on string that gets thicker
⇒ transmitted pulse has no change
⇒ reflected pulse → apply above rules





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Velocity of wave depends on medium

"denser" = slower

so at boundary where density increases:

- transmitted pulse

  Amplitude: will decrease
  Phase: no change
  Frequency: no change (doesn't depend on medium!)
  Velocity: decreases
  Wavelength: $v = \lambda f$
  so $v\downarrow$ then $\lambda\downarrow$

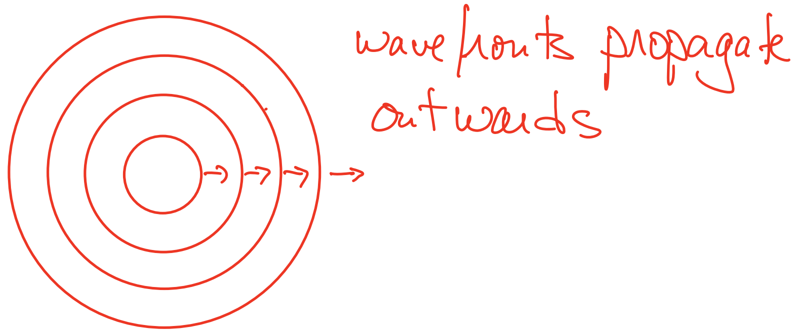
- reflected pulse

  Amplitude: decreases due to energy lost to transmitted pulse
  Phase: 0° if density in transmitted

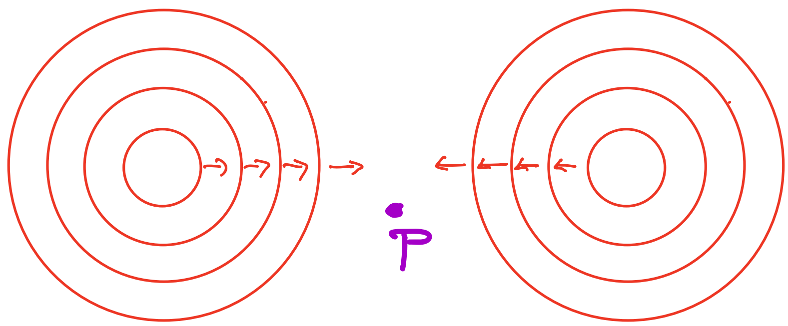
region is less
180° if density in transmitted
region is greater
Freq: always the same
Velocity: no change (same region?)
Wavelength: $\lambda = \frac{v}{f}$ so no change

Principle of superposition

Throw stone in a pond → makes a wave



Throw another stone in at different location



At some point "P", both waves will cause vertical displacement

Principle of superposition: waves add linearly

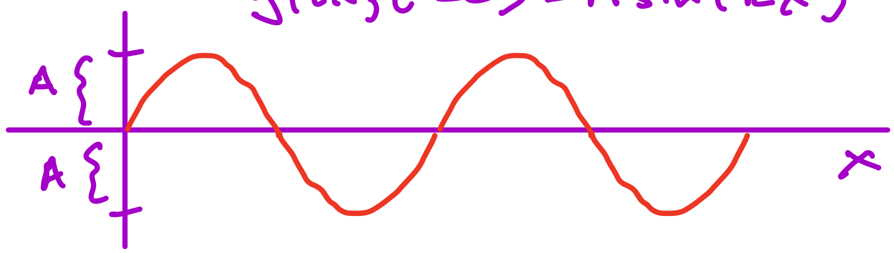
so $y(P) = y_1(P) + y_2(P)$

ex: traveling wave in 1-D

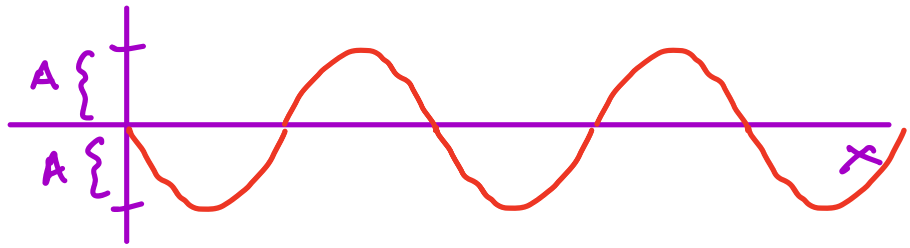
wave function: $y_1 = A\sin(kx - wt)$

here is what the wave looks like at $t=0$

$y_1(x, t=0) = A\sin(kx)$



take a second wave w/ same A, k, T
but with a phase offset



$y_2(x,t) = A\sin(kx - \omega t + \phi)$

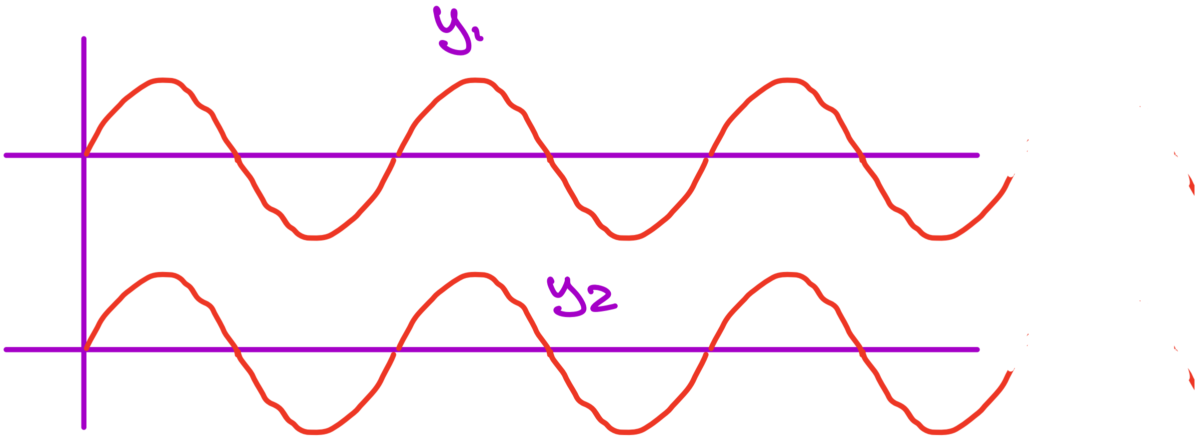
at $t=0$, $y_2(x=0, t=0) = A\sin\phi = 0$

so $A\sin\phi = 0 \Rightarrow \phi = \pi$

now add y_1 & y_2

$\Rightarrow$ can ignore "ωt" part $\rightarrow$ same for both

(think of it as adding the same thing to each phase) set $\phi = 0$ (no difference in phase)



principle of superposition: add linearly

$y_{tot} = y_1 + y_2 = A\sin(kx - \omega t) + A\sin(\;\;)$

resulting wave $y_{tot} = 2A\sin(kx - \omega t)$

same f, λ but $2A$ amplitude

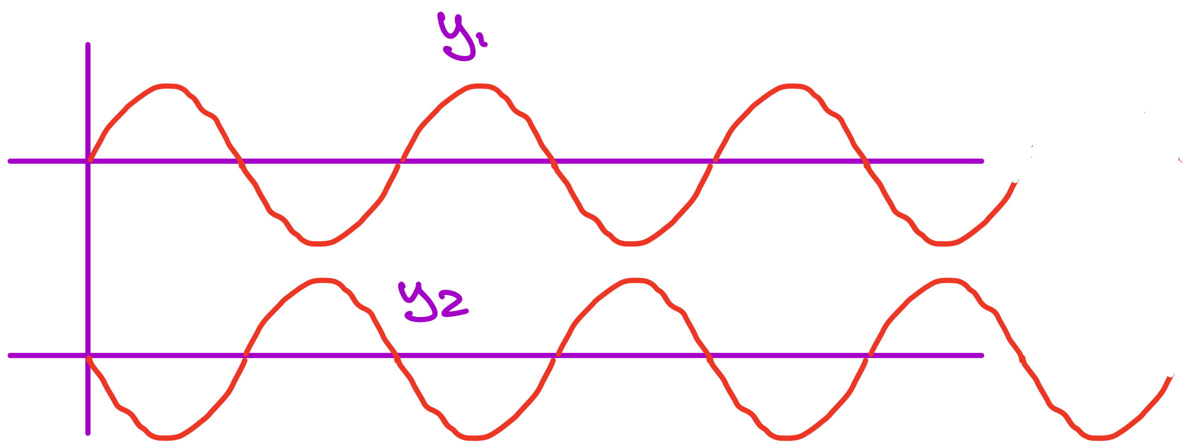
but since power ~ (amp)^2, $P_{tot} = 4P_{1\text{ or }2}$

$\Rightarrow$ this is called "constructive interference"

note: if $\phi = 0$, or 2π, same thing

constructive: relative phase $\phi = 2n\pi$, $n = 0, 1, 2 \ldots$

next set $\phi = \pi$



y_{tot} will be ϕ everywhere!

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this is "destructive interference"

destructive : $\phi = \pi, 3\pi, 5\pi \ldots$

$= n\pi$, $n = 1, 3, 5 \ldots$ odd int

or $= (2n+1)\pi$, $n = 0, 1, 2 \ldots$ any int

see https :www.physics.umd.edu/hep/drew/shiva_wave.html#superposition

for more on adding waves together

Superposition w/ only phase shifts

let $y_1 = A\sin(kx - \omega t + \phi)$ } same A, T, λ
$y_2 = A\sin(kx - \omega t)$

$$y_{tot} = A\sin(kx-\omega t+\phi) + A\sin(kx-\omega t)$$

write $A\sin(kx-\omega t+\phi) = A\sin(kx-\omega t+\frac{1}{2}\phi+\frac{1}{2}\phi)$
$A\sin(kx-\omega t) = A\sin(kx-\omega t+\frac{1}{2}\phi-\frac{1}{2}\phi)$

let $a \equiv kx-\omega t+\frac{1}{2}\phi$
then $y_1 = A\sin(a+\frac{1}{2}\phi)$
$y_2 = A\sin(a-\frac{1}{2}\phi)$

remember: $\sin(x\pm y) = \sin x\cos y \pm \cos x \sin y$

$$\begin{aligned} y_1+y_2 &= A\sin(a+\tfrac{1}{2}\phi) + A\sin(a-\tfrac{1}{2}\phi) \\ &= A[\sin(a)\cos(\tfrac{1}{2}\phi) + \cos(a)\sin(\tfrac{1}{2}\phi)] \\ &\quad + A[\sin(a)\cos(\tfrac{1}{2}\phi) - \cos(a)\sin(\tfrac{1}{2}\phi)] \\ &= 2A\sin(a)\cos(\tfrac{1}{2}\phi) \end{aligned}$$

so $y_{tot} = \underbrace{2A\cos(\tfrac{1}{2}\phi)}_{\text{new amplitude}}\sin(\underbrace{kx-\omega t+\tfrac{1}{2}\phi}_{\text{resulting wave is phase shifted by } \frac{1}{2}\phi})$

y_{tot} has same time dependence

- amplitude $2A\cos(\frac{1}{2}\phi)$
- shifted $\frac{1}{2}\phi$ ("average" phase shift)

ex: $\phi = \pi$

y_{tot} amplitude $2A\cos\frac{1}{2}\pi = 0$ destructive!

$\phi = 2\pi$ $\quad 2A\cos\frac{1}{2}2\pi = -2A$

shifted by π

$$y_{tot} = 2A\cos(\pi)\sin(kx - \omega t + \pi)$$

write $\sin(kx-\omega t+\pi) = \sin(kx-\omega t)\underbrace{\cos\pi}_{-1}$
$\quad + \cos(kx-\omega t)\underbrace{\sin\pi}_{0}$

$= 2A\sin(kx-\omega t)$ ✓

Standing Waves

Take 2 traveling waves:

- Same A amplitude
- same λ wave length & f freq
- opposite directions

$$y_1 = A\sin(kx - \omega t)$$

$$y_2 = A\sin(kx + \omega t)$$

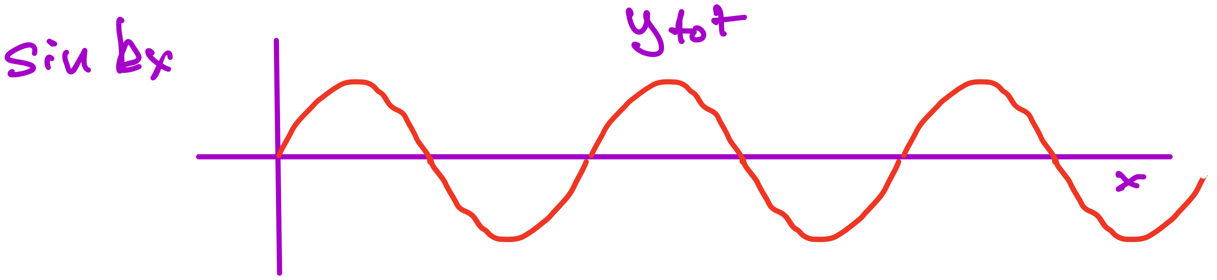
$$y_{tot} = y_1 + y_2 = A\sin(kx - \omega t) + A\sin(kx + \omega t)$$

use $\sin(a \pm b) = \sin a \cos b \pm \cos a \sin b$

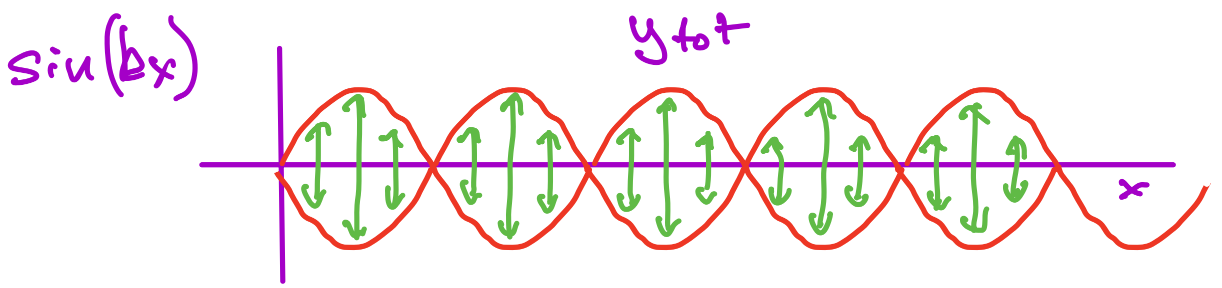
so $y_{tot} = A[\sin kx \cos\omega t - \cos kx \sin\omega t$
$\quad + \sin kx \cos\omega t + \cos kx \sin\omega t]$

$= 2A \sin kx \cos\omega t$

$$= \underbrace{(2A\cos\omega t)}_{\text{amplitude}} \cdot \underbrace{\sin kx}_{\text{periodic in space}}$$



but "amplitude" at each point is oscillating in time



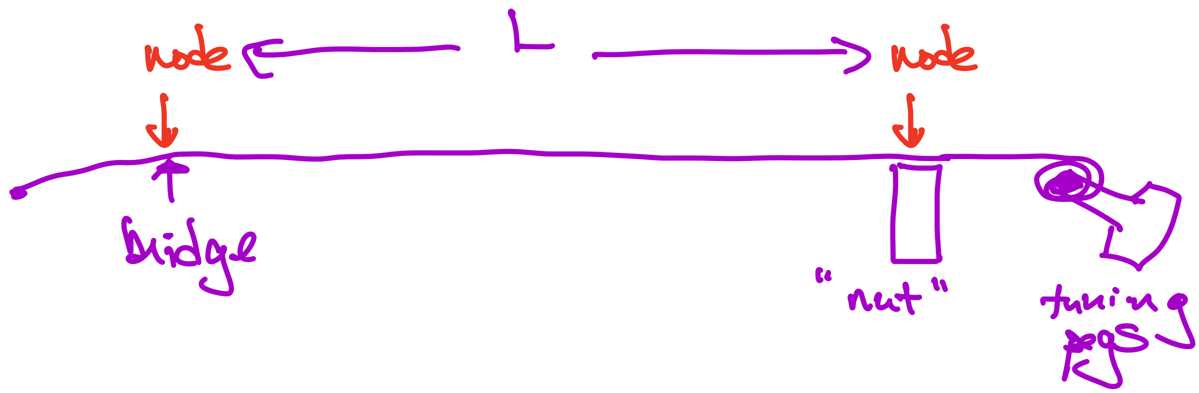
this is called "standing wave"

see https://www.physics.umd.edu/hep/drew/shiva_wave.html#superposition

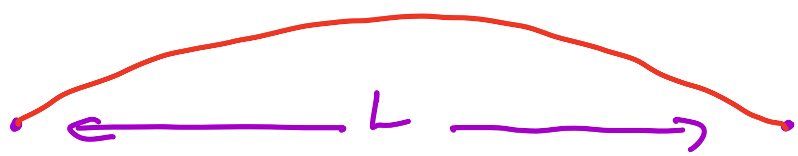
and set velocities to be equal & opposite

traveling waves: can come from waves reflecting and interfering

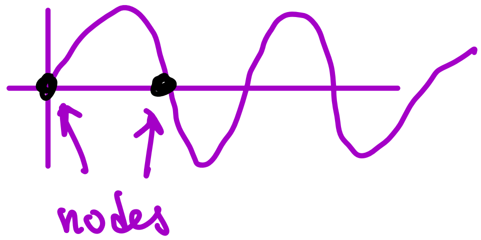
e.g. waves on musical instrument
but only for frequencies that produce reflected waves and both have nodes at boundaries



lowest frequency that will fit will have nodes at both ends = wavelength = 2L



for sin wave:



so $L = \frac{1}{2}\lambda_1$ or $\lambda_1 = 2L$

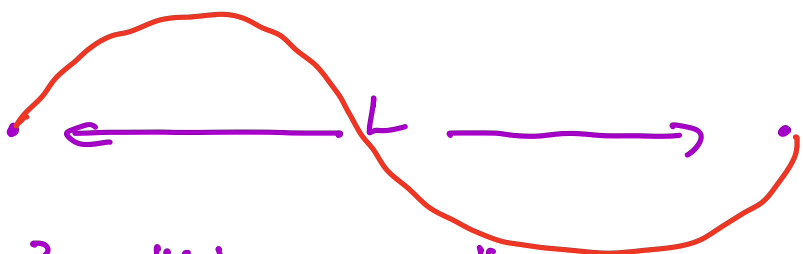
λ_1 is called "fundamental

$f_1\lambda_1 = v$ which is a property of string

$$v = \sqrt{T/\mu}$$

so $f_1 = \frac{v}{\lambda_1} = \frac{v}{2L}$

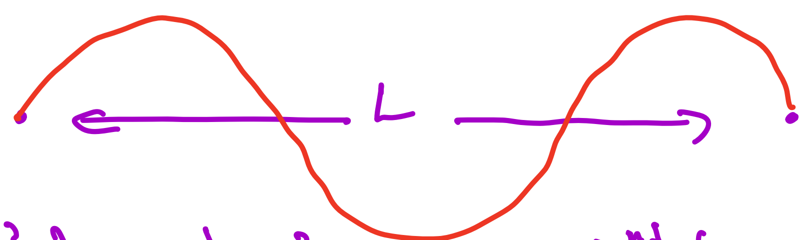
next:



$L = \lambda_2 \Rightarrow \lambda_3 = L = \frac{2}{2}L$ "1st harmonic"

$$f_2 = \frac{v}{\lambda_2} = \frac{v}{L} = \frac{2v}{2L} = 2f_1$$

next



$L = \frac{3}{2}\lambda_3 \Rightarrow \lambda_3 = \frac{2}{3}L$ 2nd harmonic

$$f_3 = \frac{v}{\frac{2}{3}L} = \frac{3v}{2L} = 3f_1$$

$\vdots$

$$\left.\begin{aligned} f_n &= n f_1 \\ \lambda_n &= \frac{2L}{n} \end{aligned}\right\} \quad n = 1, 2, \ldots$$

these are called "normal modes" of the oscillations of the string

and f_n are the "normal frequencies"

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